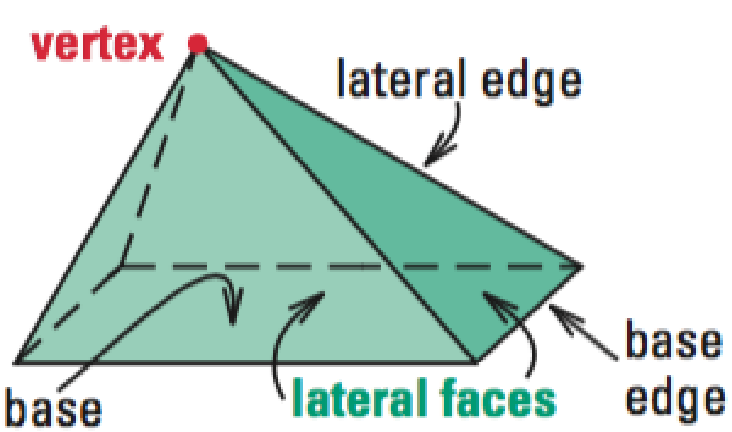
Chapter 12
Surface Area and Volume

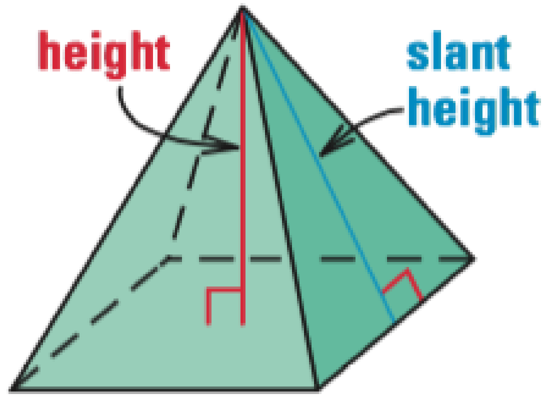
Section 3
Surface Area of Pyramids and Cones

GOAL 1: Finding the Surface Area of a Pyramid

A **pyramid** is a polyhedron in which the *base* is a polygon and the *lateral faces* are triangles with a common *vertex*. The intersection of two lateral faces is a *lateral edge*. The intersection of the base and a lateral face is a *base edge*. The *altitude*, or *height*, of the pyramid is the perpendicular distance between the base and the vertex.



Pyramid

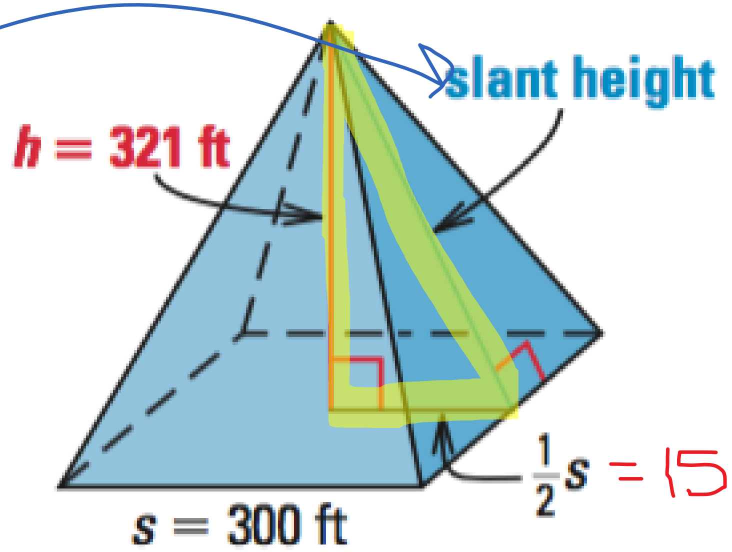


Regular pyramid

A **regular pyramid** has a regular polygon for a base and its height meets the base at its center. The *slant height* of a regular pyramid is the altitude of any lateral face. A nonregular pyramid does not have a slant height.

Example 1: Finding the Area of a Lateral Face

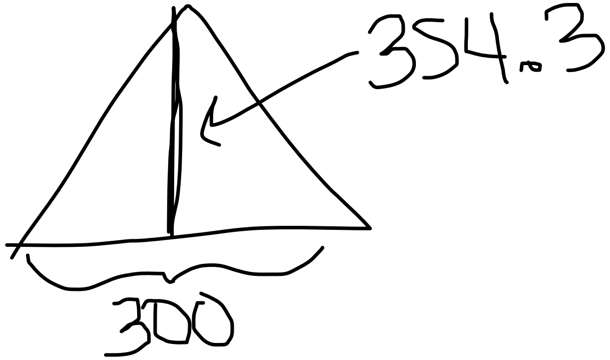
ARCHITECTURE The lateral faces of the Pyramid Arena in Memphis, Tennessee, are covered with steel panels. Use the diagram of the arena at the right to find the area of each lateral face of this regular pyramid.



$321^2 + 150^2 = \ell^2$

$\sqrt{125541} = \sqrt{\ell^2}$

$354.3 = \ell$

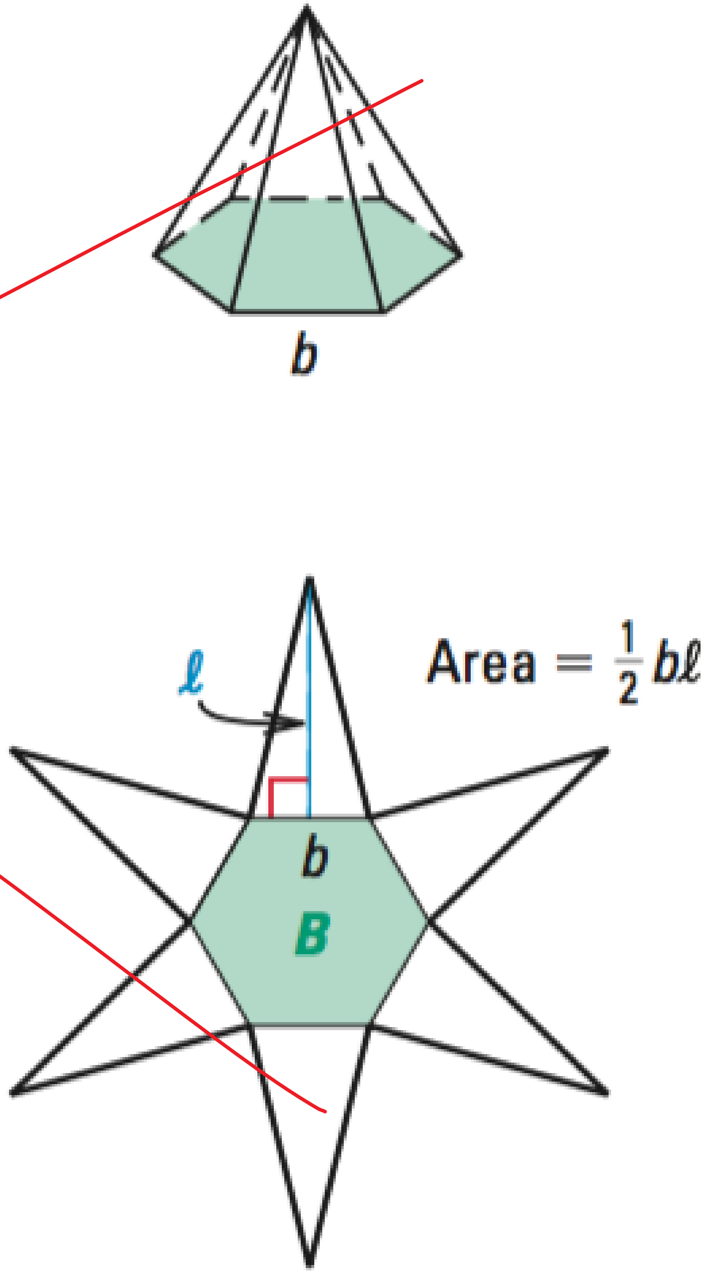


$\frac{1}{2}bh$

$\frac{1}{2}(300)(354.3)$

$53{,}145\ ft^2$

A regular hexagonal pyramid and its net are shown at the right. Let b represent the length of a base edge, and let ℓ represent the slant height of the pyramid.



The area of each lateral face is $\frac{1}{2}b\ell$ and the perimeter of the base is $P = 6b$. So, the surface area is as follows:

$$S = (\text{Area of base}) + 6(\text{Area of lateral face})$$

$$S = B + 6\left(\frac{1}{2}b\ell\right) \qquad \textbf{Substitute.}$$

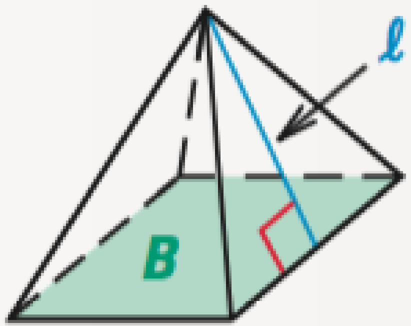
$$S = B + \frac{1}{2}(6b)\ell \qquad \textbf{Rewrite } 6\left(\frac{1}{2}b\ell\right) \textbf{ as } \frac{1}{2}(6b)\ell.$$

$$S = B + \frac{1}{2}P\ell \qquad \textbf{Substitute } P \textbf{ for } 6b.$$

THEOREM

THEOREM 12.4 ***Surface Area of a Regular Pyramid***

The surface area S of a regular pyramid is $S = B + \frac{1}{2}P\ell$, where B is the area of the base, P is the perimeter of the base, and ℓ is the slant height.



Rect.: $(b \times h) + \frac{1}{2}P\ell$

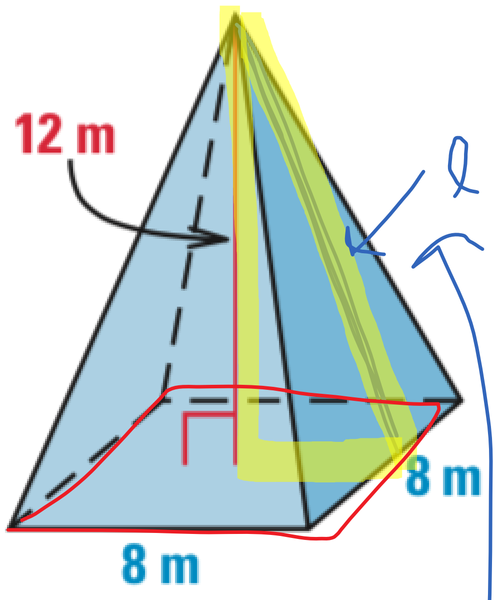
Tri.: $(\frac{1}{2}bh) + \frac{1}{2}P\ell$

Example 2: Finding the Surface Area of a Pyramid

$B + \frac{1}{2}Pl$

$\uparrow$

$(b \times h)$



$(8 \times 8) + \frac{1}{2}(8+8+8+8)(12.6)$

$4^2 + 12^2 = l^2$

$\sqrt{160} = \sqrt{l^2}$

$12.6 = l$

$64 + 201.6$

$265.6\ m^2$

Example 2: Finding the Surface Area of a Pyramid

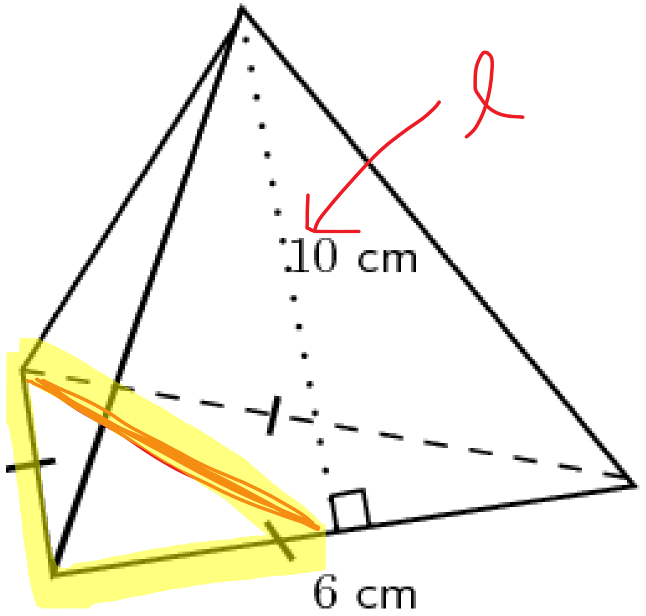
$B + \frac{1}{2}P\ell$

$(\frac{1}{2}bh)$

$(\frac{1}{2} \times 6 \times 5.2) + \frac{1}{2}(6+6+6)(10)$

$15.6 + 90$

105.6 cm^2



$3^2 + x^2 = 6^2$

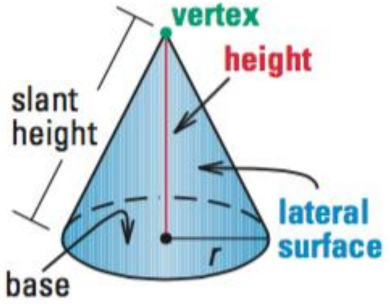
$9 + x^2 = 36$

$\sqrt{x^2} = \sqrt{27}$

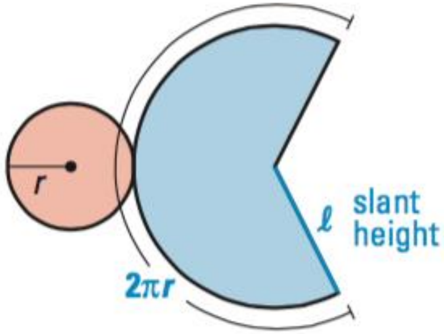
$x = 5.2$

GOAL 2: Finding the Surface Area of a Cone

A **circular cone,** or **cone,** has a circular *base* and a *vertex* that is not in the same plane as the base. The *altitude*, or *height*, is the perpendicular distance between the vertex and the base. In a **right cone,** the height meets the base at its center and the *slant height* is the distance between the vertex and a point on the base edge.



The **lateral surface** of a cone consists of all segments that connect the vertex with points on the base edge. When you cut along the slant height and lie the cone flat, you get the net shown at the right. In the net, the circular base has an area of πr^2 and the lateral surface is the sector of a circle. You can find the area of this sector by using a proportion, as shown below.



$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Arc length}}{\text{Circumference of circle}}$$ **Set up proportion.**

$$\frac{\text{Area of sector}}{\pi \ell^2} = \frac{2\pi r}{2\pi \ell}$$ **Substitute.**

$$\text{Area of sector} = \pi \ell^2 \cdot \frac{2\pi r}{2\pi \ell}$$ **Multiply each side by $\pi \ell^2$.**

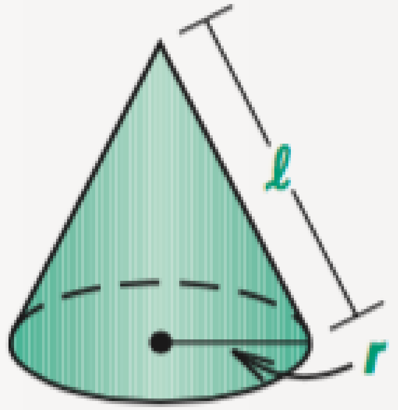
$$\text{Area of sector} = \pi r \ell$$ **Simplify.**

The surface area of a cone is the sum of the base area and the lateral area, $\pi r \ell$.

THEOREM

THEOREM 12.5 ***Surface Area of a Right Cone***

The surface area S of a right cone is $S = \pi r^2 + \pi r\ell$, where r is the radius of the base and ℓ is the slant height.



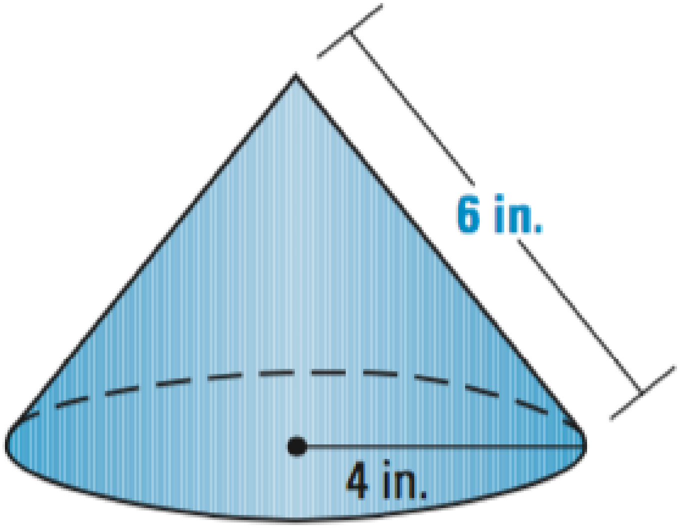
Example 3: Finding the Surface Area of a Right Cone

$\pi r^2 + \pi r \ell$

$3.14(4^2) + 3.14(4)(6)$

$50.24 + 75.36$

125.6 in^2



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